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SCHOOL OF EDUCATION
INDIANA UNIVERSITY

Teachers' and Students' Understanding of Arithmetic

by
LEWIS W. STONEKING
and
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PREFACE

This bulletin presents an instrument developed to measure selected basic arithmetical principles and generalizations. In addition it presents a study of the use of the instrument with students in grades 8 and 12, undergraduate students in grades 14, 15, and 16 taking a teacher preparation program, and teachers in seven geographic locations in the state of Indiana.

For 30 years or more there has been much writing and discussion at all educational levels concerning the need for improving the quality of instruction in arithmetic. Although no sudden metamorphosis has taken place, promising signs of improvement in teaching arithmetic are to be found at all levels of the elementary school. Continuing improvement is certain to take place, even though it will probably be somewhat uneven. One of the underlying reasons for this uneven improvement is the inadequate mathematical background of many in-service teachers.

There was a time when it was thought that arithmetic could be taught by any teacher with a little preparation beyond high school. It was reasoned that almost anyone who had completed grades 1 through 8 could recite the multiplication tables, do long division, divide one fraction by another, and even use percentage and ratio. After all, what else would be needed to teach arithmetic to children?

With the advent of increased emphasis upon mathematical meaning and structure, it soon became clear that teachers needed to know much more than rote, computational processes. Arithmetic must be taught as a system of related processes, ideas, and principles. The implication is clear. Teachers must understand basic arithmetical principles and generalizations. Only then will teachers be able to help children develop those concepts essential for providing a solid foundation for quantitative thinking and for later mathematical learning.

Ronald C. Welch

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L.W.S.
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VITAE

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TEACHERS' AND STUDENTS' UNDERSTANDING OF ARITHMETIC

BACKGROUND FOR THE STUDY

Many authorities on the teaching of arithmetic have written books and articles about the necessity for children understanding arithmetic, and they have indicated that a function of the elementary teacher is that of helping children elicit generalizations, or principles, from their arithmetic experiences. If teachers are to teach effectively, it seems necessary for them to know basic arithmetical principles, generalizations, or relationships.

The importance of meaning in arithmetic. The history of mathematics reveals that in ancient times the Greeks and the Romans had high regard for arithmetic, known as the science of numbers, and lesser emphasis was given to computation with numbers. At that time the emphasis was on studying the generalizations and relationships. About the beginning of the sixteenth century arithmetic began to be recognized as a tool subject, and arithmetic as the science of numbers began to decline.

As a skill and a tool subject only, arithmetic fit beautifully into early theories of learning, such as those set forth by the Faculty Psychologists, Associationists, and Connectionists. These theories indicated that learning arithmetic facts and processes strengthened portions of the brain or formed bonds or connections between stimulus and response. Thus for many decades arithmetic was thought to be made up of a number of comparatively unrelated facts, skills, abilities, and bits of knowledge. Teachers believed that students learned first by following the teacher's example and then by repeatedly drilling on the acquired response. All students were required to master facts, skills, abilities, and bits of knowledge whether they understood them or not. Rote drill and memorization had supplanted the idea of arithmetic as the science of numbers.

Although many people adhered to the practices advocated by the earlier learning theories, there were some who did not and who sharply criticized rote drill techniques. These dissenters believed that the restricted pattern of teaching subscribed to by these early theories of learning lowered, rather than raised, the student's power to learn; that reasoning was subordinated to repetition; and that teachers using the drill method replaced intellectual activity with rote memorization of rules and procedures. The drill method was believed to be nothing more than blind performance of tricks, with the dignification of mathematical terminology.

The overemphasis on drill that characterized the learning of arithmetic in the 1920's has been definitely reduced. That method of teaching is now generally out of favor with educational psychologists. Many authorities have turned to the Gestalt, or field, theory of learning. Field theorists believe that students begin with the whole and try to establish relationships between and among the parts and the whole. Learning is the differentiation and organization of parts into a whole, and the resulting re-organization is accompanied with new learnings.

McConnell[1] presented the relationship of the learning of elementary arithmetic to the field theory. He stated that learning is more a matter of seeing some kind of organization and meaning in the field of experience than it is of establishing connections between stimuli and responses. Field theorists recognize that helping learners elicit generalizations involves more time than rote memorization, but they believe that, if pupils establish relationships, they will develop a firmer grasp of arithmetic and will be better able to work related problems. They believe that students will develop the notion that arithmetic is more a matter of reasoning than of memorization of unrelated rules and data.

One of the earlier books emphasizing the meaningful approach was Warren Colburn's[2] Intellectual Arithmetic upon the Inductive Method. In this book he presented his argument for the development of arithmetical principles or generalizations. He indicated that the idea of quantity is first recognized through contact with concrete objects; then, when a person observes that quantity is common to all things, he begins to develop the abstract idea of quantity. First a person makes calculations with similar objects, then soon learns that the same calculations apply to dissimilar objects, and finally the learner is able to deal with abstraction.

Colburn believed that arithmetic reasoning develops upon the principle of analytic induction the same as do the other sciences, that general principles are established from specifics, and that these principles proceed from the simple to the complex.

He indicated that learners do not readily understand abstract problems and that any practice with abstract numbers is meaningless until the learner has discovered the underlying principle.

Colburn believed that students seldom understand practical problems when they first learn by means of abstract examples and that it is easier for students to understand the method of finding the result if

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[1]McConnell, T. R., "Recent Trends in Learning Theory and Their Application to Psychology of Arithmetic," in Arithmetic in General Education, p. 280.

[2]Colburn, Warren, Intellectual Arithmetic upon the Inductive Method, p. iv.

practical problems are used. Learners do not discover the meaning until they have solved several purposeful problems and have established a relationship. He recognized that it is important for students to practice with abstract problems after the understanding has developed.

The acceptance of the meaningful approach to arithmetic as a method of instruction was discussed by Brownell[3] in 1945. He said that it was fashionable to teach arithmetic meaningfully, or at least to say that it was meaningfully taught. The acceptance of the meaningful approach was quite contrary to beliefs of teachers in the 1920's. Each new arithmetic book printed in that period was accompanied by claims that it contained a program for making arithmetic meaningful. It should be parenthetically noted that in the 1960's many arithmetic books were accompanied by claims that they presented the modern, fundamental, or basic concept approach to mathematics.

In the same article Brownell[4] pointed out that theoretically there was agreement about teaching arithmetic meaningfully, but there was not agreement in practice. Some advocates of the meaningful approach to arithmetic disregarded, or at least minimized, the importance of arithmetical meanings in favor of social meanings. They believed that gaining experiences in using the skills in arithmetic made them meaningful. Brownell believed that pupils using skills may develop awareness of number usefulness (this he called significance) but that this could not develop number meaning. Meaning must be found in the organization, the structure, the inner relationships of arithmetic itself.

One of the most quoted mathematics authorities, Brownell,[5] said that, although arithmetic has been taught in the past in a nonmathematical manner, it is important to consider arithmetic a vital part of the mathematical program. He further stated that teachers would have to change their teaching practices drastically if they would agree not to teach any arithmetical idea, skill, or process unless it could be taught sensibly. If this were done, then teachers would discover that they would have to teach arithmetic mathematically, for arithmetic is an inseparable part of mathematics.

Recently McSwain[6] said that it is important for children to discover arithmetic meanings and to learn to think quantitatively. The

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[3]Brownell, W. A., "When Is Arithmetic Meaningful?" Journal of Educational Research 38:481, March, 1945.

[4]Ibid., p. 481.

[5]Brownell, W. A., "Essential Mathematics for Minimum Army Needs," School Review 52:484-492, October, 1944.

[6]McSwain, E. T., "Discovering Meanings in Arithmetic," Childhood Education 26:267, February, 1950.

function of the teacher is to guide pupils' search for the generalizations that underlie the decimal number system and the operational processes by providing classroom situations and materials that will stimulate questions about quantity and relationships of quantity.

Arithmetical experiences should be systematically and sequentially organized so as to stimulate the learner to proceed through the various levels of abstraction to the point where they are able to elicit the basic arithmetical principles.

One of the major contributions of the last quarter-century may prove to be the practice of carrying learning to the level of abstraction that permits the understanding of the relationships and systematic character of arithmetic. This emphasis on generalizations has not conflicted with the practice of introducing arithmetical topics through concrete experiences.

Many authorities firmly believe that arithmetic instruction should involve students in eliciting arithmetical generalizations from their experiences. One of the functions of the elementary teacher is to provide opportunities for students to discover the basic principles that underlie the computational processes and those principles that are essential to understanding our number system.

Children's understanding of basic arithmetical principles and generalizations. The existing evidence dealing with elementary and secondary students' understanding of basic arithmetical principles and generalizations seems to indicate that their understanding appears to be as meager as the evidence. It would seem safe to say that almost no research has been done in this area.

Glennon[7] was the first to test students' knowledge of basic arithmetical meanings. He designed an 80-item test that measured understanding of the decimal system of notation, integers and processes, fractions and processes, and decimals and processes. He found that seventh, eighth, ninth, and twelfth grade students made very low scores (ranging from 12 to 37 per cent), and concluded that arithmetical meanings were not being learned.

Rappaport[8] designed a 72-item experimental test of the basic concepts and processes of arithmetic and tested seventh and eighth grade students in nine elementary schools. He found that the students did not have an adequate understanding of the meanings of arithmetic and concluded that computational skill is not a good index of the understanding of meanings of the processes used in computation.

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[7]Glennon, V. J., "Testing Meanings in Arithmetic," in Arithmetic 1949, pp. 70-74.

[8]Rappaport, David, An Investigation of the Degree of Understanding of Meanings in Arithmetic of Pupils in Selected Elementary Schools, 167 pp.

Feinstein[9] devised a 40-item test of the meanings of fractions and administered it to 418 sixth- and seventh-grade students. He found that students lack understanding of the basic meanings of fractions and that computational skill is a poor index of the level of understanding of meanings.

It is interesting to note that both Rappaport and Feinstein found that computational skill is not indicative of understanding of basic arithmetical principles.

The results of a study by Dutton[10] revealed that the students themselves believed that one of the basic reasons for their dislike of arithmetic was a lack of understanding of it.

Certainly it is evident that elementary teachers should guide students' acquisition of the abstractions that are fundamental to an understanding of arithmetic, and that students seem to lack understanding of basic arithmetical principles and generalizations. One possible cause of this situation may be that teachers themselves have acquired a less than adequate understanding of basic mathematical generalizations and principles.

Teachers' background and understanding of basic arithmetical principles. Research on teachers' understanding of arithmetic and on the preparation of teachers is of limited quantity, but the data that are available indicate that this topic is a matter for serious concern.

One study by Glennon[11] revealed that the average raw score of college seniors on an 80-item test of basic mathematical understandings was one point less than the average raw score of freshmen, and that both groups understood less than half the mathematical understandings that are basic to the computational processes taught in the first six grades of the elementary school.

Similar data were reported by Phillips,[12] who gave a 100-item arithmetic test to prospective teachers at the University of Illinois.

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[9]Feinstein, I. K., An Analytic Study of the Understandings in Common Fractions Possessed by a Selected Group of Sixth and Seventh Grade Pupils, 127 pp.

[10]Dutton, W. H., "Attitudes of Junior High School Pupils Toward Arithmetic," The School Review 64:18-22, January, 1956.

[11]Glennon, V. J., A Study of the Guidance and Growth and Mastery of Certain Mathematical Understandings on Seven Educational Levels, 189 pp.

[12]Phillips, Clarence, "Background and Mathematical Achievement of Elementary Education Students in Arithmetic for Teachers," School Science and Mathematics 53:48-52, January, 1953.

He found that they made a mean score of 72 per cent, while the published norm for eighth-grade students was 62 per cent.

Newsom[13] said that elementary school teachers are frequently only a step ahead of their bright students. He revealed the fact that some elementary teachers had told him they were not confident when working with their classes on arithmetical concepts.

Schaaf[14] reported that many individuals who are planning to teach or are now teaching are not able to verbalize clearly their thoughts when trying to explain concepts and processes of arithmetic. Schaaf believed that arithmetic teaching is frequently so routinized that understandings are lost in the process. He hypothesized that a general lack of understanding on the part of the teachers of the relationships, abstractions, or arithmetical generalizations is the result of the teachers' teaching the short-cut as a process rather than teaching the process itself. He was definite in stating that the lack of understanding is not due to some inherent difficulty of arithmetic, but that the lack of understanding is probably a function of the way in which arithmetic is taught. Schaaf said that many teachers have expressed a lack of knowledge of the fundamental understandings and processes and that they are not able to use arithmetic meaningfully and critically.

Another aspect of the problem was brought to light by Newsom,[15] who was of the opinion that one possible cause of the failure of some elementary schools to develop understandings is that the teachers who teach elementary teachers are frequently unprepared for the job.

In many school systems, in the present mathematics curriculum arithmetic is taught only at the elementary school level. The high school mathematics curriculum is not unlike the curriculum of a half dozen decades ago--essentially algebra, geometry, trigonometry, more algebra, and perhaps solid geometry or advanced algebra. This pattern is frequently repeated on the college level.

Buswell[16] indicated there are few high schools which offer courses in advanced arithmetic and few institutions of higher learning which offer advanced arithmetic on a scholarly level. Many elementary teachers are finishing their academic preparation with no more training in arithmetic than they had at the end of the eighth grade, and by the time they finish college at least an eight-year period of forgetting has taken place.

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[13]Newsom, C. V., "Mathematical Background Needed by Teachers," in The Teaching of Arithmetic, p. 237.

[14]Schaaf, W. L., "Arithmetic for Arithmetic Teachers," School Science and Mathematics 53:537, October, 1953.

[15]Newsom, op. cit., p. 236.

[16]Buswell, G. T., "Arithmetic," in Encyclopedia of Educational Research, p. 73.

Weaver[17] pointed out that few prospective elementary teachers take elective courses in mathematics. He stated that one of the problems of preparing elementary school teachers comes from the low level of understanding of arithmetic that is found among students in colleges and universities.

Wilburn and Wingo[18] noted that people study arithmetic only in the elementary school, not in high school or college, and that mastery of arithmetic on the elementary level is incomplete. They believed that one of the possible reasons that very little mathematics is studied above the elementary level is that arithmetic is taught so poorly in the elementary school.

Grossnickle[19] surveyed 131 of the 147 colleges of the American Association of Colleges for Teacher Education and found that 76 per cent of the colleges did not require any high school mathematics course for entrance. He found that less than half of the colleges offered a background mathematics course for prospective teachers, and in these colleges it was offered for an average of 1.5 semester hours of credit for advanced-grade teachers and 1.2 semester hours for intermediate-grade teachers.

Dyer[20] assumed, in the absence of data, that a teacher with a thorough understanding of the basic arithmetical principles is better able to develop a thorough understanding in his students than a teacher with a less thorough understanding. This assumption led to the conclusion that, since the limited available research seems to indicate that this understanding is not present in teachers, future teachers will go through the elementary schools not learning basic arithmetic, but learning to dislike mathematics. These future elementary teachers will drop it as soon as possible in high school and will disregard it in teachers college, since it is not required; they will then return to the elementary school and teach another generation to dislike arithmetic.

Weaver[21] believes that it will not be possible for future teachers to teach many of the basic and important understandings, since they do not know them and are not learning them in teacher training institutions at the present time.

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[17]Weaver, J. F., "A Crucial Problem in the Preparation of Elementary Teachers," Elementary School Journal 56:255-261, February, 1956.

[18]Winburn, D. B., and Wingo, G. M., "In-Service Development of Teachers of Arithmetic," in The Teaching of Arithmetic, p. 254.

[19]Grossnickle, F. E., "The Training of Teachers in Arithmetic," in The Teaching of Arithmetic, pp. 203-231.

[20]Dyer, H. S.; Kalin, Robert; and Lord, F. M., Problems in Mathematical Education, p. 18.

[21]Weaver, op. cit., p. 257.

As has been shown, many colleges do not require any arithmetic courses for prospective teachers. Weaver[22] pointed out that the situation may be of even greater consequence than might be assumed. Simply offering required courses for students on an elementary school preparation program does not guarantee that the required arithmetic courses offered emphasize the acquisition of basic arithmetical understandings.

Recommended basic arithmetical principles. The literature suggested a number of basic arithmetical principles with which teachers and prospective teachers should be familiar.

The National Council of Teachers of Mathematics[23] indicated that one of the main requirements for prospective elementary teachers is an understanding of arithmetic. It was postulated that teachers must develop the ability to teach the basic concepts, processes, and vocabulary of arithmetic; the decimal system of numeration, which includes the concept of decimal fraction; computation of whole numbers; common and decimal fractions; and several other understandings.

An opinion expressed by William Schaaf[24] indicates that in nothing taught in the elementary school is it more important that the teachers have an adequate understanding of content itself than in the area of arithmetic. He suggested that elementary teachers probably have less insight into the content of arithmetic than into that of any other subject.

Schaaf[25] further indicated that teachers should be familiar with the historical development of numerals, numeration, number systems, computational procedures, the foundations of arithmetic, and its social applications.

Wilburn and Wingo[26] affirmed their belief in the idea that to help children learn the generalizations it is necessary for teachers to study or review such concepts as the decimal nature of the Hindu-Arabic number system and its implications for the fundamental arithmetic operations. The teachers should also know the relation of the decimal system to uses such as measurement, systems of money, and time.

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[22] Ibid., p. 258.

[23] National Council of Teachers of Mathematics, Commission on Post-War Plans, "Guidance Pamphlet in Mathematics for High School Students," Mathematics Teacher 40:315-339, November, 1947.

[24] Schaaf, op. cit., pp. 537-543.

[25] Ibid., p. 541.

[26] Wilburn and Wingo, op. cit., pp. 251-268.

It was indicated by Newsom[27] that prospective teachers should understand that the position a digit occupies in a number notation determines its value. He also stated that adequate study of the place value of digits requires a study of number bases, including bases other than ten.

Members of the Commission of the Mathematical Association of America felt that, among other knowledges, a pupil who has successfully finished his sixth grade in school should be familiar with the basic concepts, vocabulary, and processes of arithmetic, and should have an understanding of the significance of the different positions that a digit may occupy in a number, including the case of decimal fractions. In the same report they said that much attention must be given to helping the students acquire a grasp of the principles underlying the fundamental processes of arithmetic, including the generalization that the order of the factors in a product does not affect the result.

Jones[28] indicated that essentials of a computing system or a numeration system are base, place, symbols, zero, and decimal, and that essentials of a number system are the commutative principle with respect to addition, the commutative principle with respect to multiplication, the associative principle with respect to addition, the associative principle with respect to multiplication, the distributive principle, and the identity element for addition and multiplication.

Summary. Earlier learning theorists were sympathetic to the drill approach to arithmetic, while the newer learning theorists hold the view that the meaningful approach is preferable.

Many authorities believe that students' elicitation of basic arithmetical principles is of prime importance and that a major function of teachers is to provide opportunities for pupils to discover the basic principles that undergird the computational processes and those principles that are fundamental to understanding a number system.

Research seems to indicate that elementary and secondary students lack understanding of basic arithmetical principles and generalizations, and that (although there is a limited quantity of research) teachers' lack of understanding of basic arithmetical principles and their background, in the area of basic arithmetic, is a matter for serious concern.

Many prospective teachers are completing their professional preparation with no preparation in arithmetic or mathematics beyond that acquired in the eighth grade. Many teacher preparatory institutions offer no course in the background of arithmetic.

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[27] Newsom, op. cit., p. 237.

[28] Jones, P. S., Understanding Numbers: Earliest Numbers; Jones, P. S., Understanding Numbers: Base and Place; Jones, P. S., Understanding Numbers: Fundamental Operations.

The literature suggests a number of basic arithmetical principles with which teachers and prospective teachers should be familiar. Among these are place value, base of a number system including bases other than ten, decimal, zero, symbols, the commutative principle with respect to addition and multiplication, the associative principle with respect to addition and multiplication, the distributive principle, and the identity element with respect to addition and multiplication.

METHODS AND PROCEDURES USED IN THE STUDY

The study[29] that served as the basis for this bulletin was concerned with determining selected factors that contributed to teachers' understanding of selected basic arithmetical principles.

The various aspects of the problem were statistically tested by the following null hypotheses:

Hypothesis 1. There is no significant difference between various age groups in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 2. There is no significant difference between groups with various amounts of teaching experience in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 3. There is no significant difference between groups at different levels of academic preparation in their understanding of selected basic arithmetical principles and generalizations.

Basic Assumptions

This investigation was based on the following assumptions:

1. Five essentials of our numeration or computing system are place value, base, decimal point, zero, and symbols; and the development of understanding of these basic principles is an essential part of arithmetic instruction.

2. Four essentials of our number system are the commutative principle with respect to addition and multiplication, the associative principle with respect to addition and multiplication, the distributive principle, and the identity element; and the development of understand-

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[29] Stoneking, L. W., Factors Contributing to Selected Basic Arithmetical Principles and Generalizations, 170 pp.

ing of these basic principles is an essential part of arithmetic instruction.

3. Two essential operations in a computing system are addition and multiplication, and subtraction and division are the inverse of these operations.

4. The judges were equally well informed on the subject of mathematical understandings.

5. The combined opinions of the panel of expert judges determined that the test was valid.

6. A correct response to an item on the test implied understanding of the principle to the level of abstraction to which the principle was measured by the specific test item; and an incorrect response, or no response, to an item on the test implied a lack of understanding of the principle to the level of abstraction to which it was measured by the specific test item.

7. The examinees chosen on the various levels were representative of other groups on the same levels located in the same areas.

8. Reading difficulties within the test items were minor, and any inadequacies in wording of individual questions were not significant.

9. Written computation on the test was held to a minimum.

10. A higher test score indicated a higher level of understanding of the selected basic arithmetical principles and generalizations.

Delimitations of the Study

This study dealt only with the positive real number system. It was restricted to five essentials of our numeration system, including place value, base, decimal point, zero, and symbols. It was also restricted to four basic principles of our number system: the commutative principle with respect to addition and multiplication, the associative principle with respect to addition and multiplication, the distributive principle, and the identity element for addition and multiplication.

The population tested in this study was restricted to a total population of 1,066 examinees. The groups selected were students in grades 8 and 12, students on a teacher preparatory program in grades 14, 15, and 16, and classroom teachers.

This study was designed to discover the understanding of the basic arithmetical principles as they were measured by the specific test items and was not designed to discover how complete the examinees' understanding was of all mathematical principles.

Procedures and Instruments Used

The procedures used in this study were selected as desirable in light of the scope of the study, the source of the data, and the instrument to be used.

1. The scope of the study was limited to the state of Indiana. It was further confined to pupils in grades 8 and 12 in a relatively large community, undergraduate students in grades 14, 15, and 16, who were taking a teacher preparatory program and teachers in seven geographic locations in the state.

The reason for selecting the various levels was that the subjects at these levels had reached certain definite stages in their study in mathematics. Those in grade 8 had completed their arithmetic (elementary mathematics) preparation but had not begun their secondary mathematics program; those in grade 12 had completed their secondary mathematics preparation; those in grade 14 were taking the elementary teacher preparation program and had completed their undergraduate junior division preparation; those in grade 15 had completed their professional training, with the exception of student teaching; and those in grade 16 had completed their undergraduate teacher preparation program. The classroom teachers who were subjects served as a basis for determining the achievement and growth of examinees as a result of age, varying numbers of years of teaching experience, and varying amounts of additional academic preparation.

2. The study was concerned with obtaining data regarding the following phases of arithmetic: place value, base of a number system including bases other than ten, decimal, zero, symbols, commutative principle with respect to addition and multiplication, associative principle with respect to addition and multiplication, distributive principle, and the identity element for addition and multiplication.

3. The objectively scored multiple-choice test was used as the most appropriate[30] instrument for collecting data about understanding of basic arithmetical principles and generalizations. Since research showed that computational skill was not a determiner of understanding of basic arithmetical principles, written computational skill was eliminated so far as possible.

4. A personal data information blank was used as the most appropriate instrument for collecting data about the examinees' experiential background. The basis upon which conclusions and implications were made was provided by the collation of the test results and by information obtained from personal data information sheets.

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[30]See Travers, R. M., How to Make Achievement Tests, p. 62; and Furst, Edward, Constructing Evaluation Instruments, pp. 251-252.

5. The scores obtained from the Test of Selected Basic Arithmetical Principles and Generalizations and data from the personal data information sheet were placed on IBM punch cards, making possible tabulations and comparisons of selected groups. Statistical treatment yielded sums, sums of squares, number of cases, variance, standard deviations, an analysis of variance among the various groups, and t tests between the means of each two adjacent classifications of each group. Statistical significance of the variances was determined.

To test statistically the specific aspects of this problem it was necessary to develop the following null hypotheses:

Hypothesis 1. There is no significant difference between means of various age groups in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 2. There is no significant difference between means of groups with various amounts of teaching experience in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 3. There is no significant difference between means of groups with different levels of academic preparation in their understanding of basic arithmetical principles and generalizations.

Hypothesis 4. There is no significant difference between means of groups with varying numbers of semesters of mathematics studied in their understanding of selected basic arithmetical principles and generalizations.

6. The reliability of the test was determined by Froelich's formula[31] and was found to be .92.

Pertinent to the above, the following procedures were carried out:

Since no test was available that measured the selected principles or generalizations that, according to the literature, are considered fundamental to arithmetic, it became necessary to construct a new instrument.

It was decided that computational skill should be avoided as a factor in determining success. It was recognized that one factor that could contribute to nonvalidity of a test of understanding of arithmetic could be eliminated by utilizing an instrument that minimized computational ability.

The instrument was designed so that it could be objectively scored in order that the speed and accuracy of an electronic scoring machine could be utilized. Multiple-choice items were used for this

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[31]Froelich, G. J., "A Simple Index of Test Reliability," Journal of Educational Psychology 32:385, May, 1941.

test, since authorities on test construction indicated that multiple-choice items were most applicable to an instrument of this nature.

It was next necessary to construct items that measured understanding of the selected arithmetical principles and generalizations.

The Test of Selected Basic Arithmetical Principles and Generalizations[32] that was finally developed and used in the study was revised three times before it was considered in final form.

The first instrument developed, the preliminary form, contained many items that were of a similar or parallel nature. This form was submitted to 14 educators in the fields of mathematics and education for their suggestions and criticisms. As a result of this procedure some items were reworded, rewritten, or eliminated, and several new items were added.

The pilot study form, or first revision, a 150-item instrument, contained all of the essential characteristics of the final instrument. It was designed to help identify ambiguities in the directions to the examinees and in the wording of individual items, and to determine the level of difficulty and the discriminatory value of each item. A preliminary form of the personal data sheet was designed for use in the pilot study.

Both the pilot study form of the test and the preliminary form of the personal data sheet were administered to three groups, having a total population of 93 students, who were enrolled in a teacher preparatory program. The examinees not only completed the test but also recommended changes. All comments were given careful consideration for further refinement of the instrument. Some items were reworded and the final form of the personal data sheet was designed.[33]

The effectiveness of each individual item was ascertained by an item analysis. The results of the item analysis and examinees' comments were used as a basis for again revising the test. The items with small discriminatory values were deleted and the remaining items were grouped so that specific items concerned with similar arithmetical principles were together. The items were then ordered according to their level of difficulty, from easy to more difficult. The various groups of items concerned with selected principles were also ordered according to their composite difficulty level. The instrument at this stage of development contained 117 items.

For the second revision of the test, the authorities form, 60 items were chosen from the 117. Care was taken to select representative items concerned with all selected arithmetical principles and generalizations on all levels of difficulty and discrimination.

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[32]See Appendix B.

[33]See Appendix A for a copy of the final form.

To determine the validity, the instrument was submitted to the combined opinion of a panel of 15 authorities who were recognized leaders in the area of arithmetic. The test was judged by 11 of the 15 authorities in accordance with the instructions.

The test was revised for the third time, and the only items retained for the final instrument were those which 75 per cent of the authorities agreed were valid items for measuring the selected basic arithmetical principles and generalizations. The composite opinion of the experts was such that only four of the 60 items were judged invalid. These four were therefore deleted. All of the items retained for the final instrument were judged valid by 80 per cent or more of the authorities, and 45 of the 56 items were judged valid by more than 90 per cent of the authorities. Several suggestions, criticisms, and recommended changes were given careful consideration in preparing the final instrument. Thirteen items parallel to those in the second revision were added to the final form of the Test of Basic Arithmetical Principles and Generalizations. When completed, the final form contained 68 items.

FINDINGS

The collation of responses to the Test of Selected Basic Arithmetical Principles and Generalizations and data obtained from the personal data sheet provided the basis for statistically determining the factors that contributed significantly to 1,066 examinees' understanding of selected basic arithmetical principles and generalizations.

Age as a Factor

To test statistically Hypothesis 1, that there is no significant difference between means of various age groups in their understanding of selected basic arithmetical principles and generalizations, the following techniques were employed and findings were established.

Data in Table 9[34] were used to apply an analysis of variance among groups of examinees of various age classifications. The results of this analysis of variance are summarized in Table 1.

From an F table, when 8 and 1,057 degrees of freedom were used, a value of 3.32 was found to be significant at the .01 level of confidence. It was therefore concluded that, for the groups sampled and within the limits of this study, age seemed to be a factor in

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[34]See Appendix C.

determining understanding of the selected basic arithmetical principles and generalizations that were tested.

TABLE 1. ANALYSIS OF VARIANCE AMONG 1,066 EXAMINEES GROUPED INTO NINE AGE CLASSIFICATIONS

Source of variation	Degrees of freedom	Sum of squares	Mean square	F level	Significance level
Between the groups	8	38,341	4,792.66		
Within the groups	1,057	152,380	144.16		
Total	1,065	190,721		3.32	.01

Data in Table 9[35] were used to apply t tests between the means of each adjacent subgroup age classification to test each of eight sub-hypotheses of Hypothesis 1 statistically. The results of these t tests are summarized in Table 2.

TABLE 2. t TESTS BETWEEN THE MEANS OF ADJACENT PAIRS OF SUBGROUPS OF 1,066 EXAMINEES GROUPED INTO NINE AGE CLASSIFICATIONS

Age groups compared (in years)	Difference between two means	S.D. of d.*	Degrees of freedom	Value of t	Significance level
13-15 and 16-18	19.55	1.35	296	-14.41*	.01
16-18 and 19-21	1.52	1.36	373	1.18	N.S.*
19-21 and 22-24	.88	1.40	336	- .63	N.S.

[35]See Appendix C.

TABLE 2 (Continued)

Age groups compared (in years)	Difference between two means	S.D. of d.*	Degrees of freedom	Value of t	Significance level
22-24 and 25-30	4.26	1.41	303	3.01	.01
25-30 and 31-36	.18	1.57	258	.12	N.S.
31-36 and 37-42	.71	2.00	140	- .35	N.S.
37-42 and 43-48	1.89	2.17	102	.87	N.S.
43-48 and 49 and older	7.51	2.12	114	3.54	.01

*S.D. of d. = standard deviation of the difference.

*Negative t values indicate that older age group in the comparison had a higher mean.

*N.S. = not significant at the .01 or .05 level of confidence.

For groups tested and within the limits of this study, from a t table when appropriate degrees of freedom were used it was found that there was a statistically significant difference in understanding of selected basic arithmetical principles and generalizations between examinees in the following age groups: examinees from 13 to 15 years of age and those from 16 to 18 years of age; examinees from 22 to 24 years of age and those from 25 to 30 years of age; and examinees from 43 to 48 years of age and those 49 years of age or more. In the first of these, the difference was in favor of the older group, but in the second and third it was in favor of the younger group.

Teaching Experience as a Factor

To test statistically Hypothesis 2, that there is no significant difference between means of groups with various amounts of teaching experience in their understanding of selected basic arithmetical prin-

ciples and generalizations, the following techniques were employed and findings were established.

Data in Table 10[36] were used to apply an analysis of variance among groups of examinees with varying amounts of teaching experience. The results of this analysis of variance are summarized in Table 3.

TABLE 3. ANALYSIS OF VARIANCE AMONG 437 TEACHERS CLASSIFIED INTO THREE TEACHING EXPERIENCE GROUPS

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value	Significance level
Between the groups	2	1,020	510.09		
Within the groups	434	65,253	150.35		
Total	436	66,273		3.39	.05

From an F table, when 2 and 434 degrees of freedom were used, a value of 3.39 was found to be significant at the .05 level of confidence. It was therefore concluded that, for the groups sampled and within the limits of this study, teaching experience seemed to be a factor in determining understanding of the selected basic arithmetical principles and generalizations that were tested.

Data in Table 10[37] were used to apply t tests between the means of each pair of adjacent subgroup teaching experience classifications to test each of two subhypotheses of Hypothesis 2 statistically. The results of these t tests are summarized in Table 4.

For the groups tested and within the limits of this study, from a t table when appropriate degrees of freedom were used, it was found that there was a statistically significant difference in understanding of selected basic arithmetical principles and generalizations between teachers in only the first of the two groups studied.

[36]See Appendix C.

[37]See Appendix C.

TABLE 4. t TESTS BETWEEN THE MEANS OF ADJACENT PAIRS OF SUBGROUPS OF 437 TEACHERS GROUPED INTO THREE TEACHING EXPERIENCE CLASSIFICATIONS

Years of teaching experience compared	Difference between two means	S.D. of d.*	Degrees of freedom	Value of t	Significance level
1 or 2 and 3 to 5	3.87	1.60	241	2.42	.05
3 to 5 and 6 or more	.75	1.37	324	.54	N.S.*

*S.D. of d. = standard deviation of the difference.

*N.S. = not significant at .01 or .05 level of confidence.

Academic Classification as a Factor

To test statistically Hypothesis 3, that there is no significant difference between the means of groups at different levels of academic preparation in their understanding of selected basic arithmetical principles and generalizations, the following techniques were employed and findings were established.

Data in Table 11[38] were used to apply an analysis of variance among groups of examinees with various academic classifications. The results of this analysis of variance are summarized in Table 5.

It was therefore concluded that, for the groups sampled and within the limits of the study, academic classification (number of years in school, or type of degree held) seemed to be a factor in determining understanding of the selected basic arithmetical principles and generalizations that were tested.

[38]See Appendix C.

TABLE 5. ANALYSIS OF VARIANCE AMONG 1,066 EXAMINEES GROUPED INTO NINE ACADEMIC CLASSIFICATIONS

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value	Significance level
Between the groups	8	42,173	5,271.68		
Within the groups	1,057	148,548	140.54		
Total	1,065	190,721		3.75	.01

From an F table when 8 and 1,057 degrees of freedom were used, a value of 3.75 was found to be significant at the .01 level of confidence. It was therefore concluded that, for the groups sampled and within the limits of this study, academic classification seemed to be a factor in determining understanding of the selected basic arithmetical principles and generalizations that were tested.

Data in Table 11[39] were used to apply t tests between means of each pair of adjacent subgroup academic classifications to test each of eight subhypotheses of Hypothesis 3 statistically. The results of these t tests are summarized in Table 6.

TABLE 6. t TESTS BETWEEN THE MEANS OF ADJACENT PAIRS OF SUBGROUPS OF 1,066 EXAMINEES GROUPED INTO NINE ACADEMIC CLASSIFICATIONS

Academic classifications compared	Difference between two means	S.D. of d.*	Degrees of freedom	Value of t	Significance level
Grade 9 and Grade 12	20.53	1.29	297	-15.88*	.01 .01
Grade 12 and Grade 14	6.68	1.67	247	4.00	.01

See Appendix C.

TABLE 6 (Continued)

Academic classifications compared	Difference between	S.D. of d.*	Degrees of freedom	Value of t	Significance level
Grade 14 and Grade 15	5.78	1.96	140	- 2.95	.01
Grade 15 and Grade 16	.08	1.89	211	- .04	N.S.‡
Grade 16 and AB	7.72	1.76	238	4.37	.01
AB and AB+	3.78	1.50	378	- 2.52	.02
AB+ and MA	3.37	1.96	332	1.72	N.S.#
MA and MA+	7.41	2.49	89	- 2.98	.01

*S.D. of d. = standard deviation of the difference.

*Negative t values indicate that the higher grade in the comparison had a higher mean.

‡N.S. = not significant at the .01 or .05 level.

#N.S. = significant at .10 level.

For the groups tested and within the limits of this study, from a t table when appropriate degrees of freedom were used it was found that there was a statistically significant difference in understanding of selected basic arithmetical principles and generalizations in favor of the older group in comparisons of examinees in the following academic classifications: examinees in grade 8 and those in grade 12; examinees in grade 14 on a teacher preparatory program and those in grade 15 on a teacher preparatory program; classroom teachers with a bachelor's degree but no additional graduate work and those with a bachelor's degree plus additional graduate work; and classroom teachers with a master's degree but no additional graduate work and those with a master's degree plus additional graduate work.

Mathematics Background as a Factor

To statistically test Hypothesis 4, that there was no significant difference between the means of groups with varying numbers of semesters of mathematics studied in their understanding of selected basic arithmetical principles and generalizations, the following techniques were employed and findings were established.

Data in Table 12[40] were used to apply an analysis of variance among groups of examinees in various mathematics background classifications. The results of this analysis of variance are summarized in Table 7.

TABLE 7. ANALYSIS OF VARIANCE AMONG 1,066 EXAMINEES GROUPED INTO EIGHT CLASSIFICATIONS ACCORDING TO THE NUMBER OF SEMESTERS OF MATHEMATICS STUDIED BEYOND GRADE 8

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value	Significance level
Between the groups	7	54,211	7,744.44		
Within the groups	1,058	136,510	129.03		
Total	1,065	190,721		21.74	.01

From an F table, when 7 and 1,058 degrees of freedom were used, a value of 21.74 was found to be significant at the .01 level of confidence. It was therefore concluded that, for the groups sampled and within the limits of this study, the number of semesters of mathematics studied after grade 8 was a factor in determining understanding of the selected basic arithmetical principles and generalizations that were tested.

Data in Table 12[41] were used to apply t tests between means of each pair of adjacent subgroups, classified according to the number of semesters of mathematics studied beyond grade 8, to test seven subhy-

[40] See Appendix C.

[41] See Appendix C.

potheses of Hypothesis 4 statistically. The results of these t tests are summarized in Table 8.

TABLE 8. t TESTS BETWEEN THE MEANS OF ADJACENT PAIRS OF SUBGROUPS OF 1,066 EXAMINEES GROUPED INTO EIGHT CLASSIFICATIONS ACCORDING TO THE NUMBER OF SEMESTERS OF MATHEMATICS STUDIED BEYOND GRADE 8

Semesters of mathematics studied beyond grade 8 compared	Difference between two means	S.D. of d.*	Degrees of freedom	Value of t	Significance level
0 and 1 or 2	6.97	1.63	171	-4.27**	.01
1 or 2 and 3 or 4	3.40	1.77	233	-1.92	N.S.±
3 or 4 and 5 or 6	6.27	1.02	501	-6.12	.01
5 or 6 and 7 or 8	4.57	1.04	544	-4.36	.01
7 or 8 and 9 or 10	.20	1.52	323	- .13	N.S.#
9 or 10 and 11 or 12	.52	2.49	130	- .21	N.S.#
11 or 12 and 13 or 14	1.92	3.15	63	- .61	N.S.#

*S.D. of d. = standard deviation of the difference.

**Negative t values indicate that the higher number of semesters in the comparison had the higher mean.

±N.S. = significant at the .01 level of confidence.

#N.S. = not significant at .01 or .05 level of confidence.

For the groups tested and within the limits of this study, from a t table when appropriate degrees of freedom were used it was found that there was a statistically significant difference in understanding

of selected basic arithmetical principles and generalizations in favor of the group with the greater amount of study of mathematics in comparisons of examinees classified according to mathematics background in the following groups: examinees with eight years of study of arithmetic and those with eight years study of arithmetic plus one or two semesters of study of mathematics beyond eight years of study of arithmetic; examinees with eight years study of arithmetic plus one or two semesters of study of mathematics and those with three or four semesters of study of mathematics beyond eight years of study of arithmetic; examinees with three or four semesters of study of mathematics beyond eight years study of arithmetic and those with five or six semesters of study of mathematics beyond eight years of study of arithmetic; and examinees with five or six semesters of study of mathematics beyond eight years of study of arithmetic and those with seven or eight semesters of study of mathematics beyond eight years of study of arithmetic. There was a direct positive relationship between the number of semesters of high school mathematics studied and understanding of selected basic arithmetical principles and generalizations.

SUMMARY, CONCLUSION, IMPLICATIONS, AND RECOMMENDATIONS

Summary

Most educators believe that arithmetic should be taught meaningfully. Although "meaning" has had social connotations to some, to others it primarily has had mathematical connotations. To the latter group of educators, major functions of elementary teachers include providing opportunities for students to elicit or discover arithmetical relationships and guiding their development of understanding of generalizations fundamental to a number system and of principles basic to the computational processes. Regardless of the interpretation of "meaning," few educators would disagree that teachers should understand basic arithmetical principles and generalizations.

The existing literature and research seemed to indicate that elementary and secondary students lack understanding of basic arithmetical principles and generalizations, and that teachers' understanding of arithmetic is a matter for serious concern.

This study grew out of a desire to discover whether selected factors contribute to students' and teachers' understanding of selected basic arithmetical principles and generalizations.

The literature suggested a number of basic arithmetical principles and generalizations with which teachers and prospective teachers should be familiar. Among these were place value, base of a number system including bases other than ten, decimal, zero, symbols, the commutative principle with respect to addition and multiplication, the

associative principle with respect to addition and multiplication, the distributive principle, and the identity element for addition and multiplication.

The purpose of this investigation was to determine whether examinees' age, amount of teaching experience, level of academic preparation, or mathematics background contributed to their understanding of selected basic arithmetical principles and generalizations.

The specific aspects of this problem were statistically tested by subhypotheses of the following null hypotheses:

Hypothesis 1: There was no significant difference between examinees of various age groups in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 2: There was no significant difference between examinees with various amounts of teaching experience in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 3: There was no significant difference between examinees on various levels of academic preparation in their understanding of selected basic arithmetical principles and generalizations.

Hypothesis 4: There was no significant difference between examinees with various numbers of semesters of mathematics previously studied in their understanding of selected basic arithmetical principles and generalizations.

The scope of this study was restricted to 1,066 examinees, including pupils in grades 8 and 12, students on a teacher preparatory program in grades 14, 15, and 16, and classroom teachers in the state of Indiana.

An objectively scored multiple-choice test was developed to determine examinees' understanding of selected principles and generalizations that were basic to arithmetic. A personal data sheet was designed for gathering data that were analyzed to determine the various classifications of age, teaching experience, general academic preparation, and mathematics background.

The final instrument used in these analyses was developed with the aid of educators in the fields of mathematics and education, examinees on a teacher preparatory program, a pilot study and subsequent item analysis, and 11 authorities in the area of arithmetic. The instrument's validity was determined by the combined opinion of a panel of experts, and no item was included in the final instrument unless at least 80 per cent of the authorities agreed the item was definitely valid for measuring understanding of some selected basic arithmetical principle or generalization. The instrument's reliability was found to be .92.

Electronic computers were used to analyze the data. Analysis of variance among the groups and t tests between the means of subgroups were used to analyze the various null hypotheses statistically.

The collation of responses to the Test of Selected Basic Arithmetical Principles and Generalizations, together with information obtained from the personal data sheet, provided the basis for statistically determining the selected factors that significantly contributed to examinees' understanding of the selected basic arithmetical principles and generalizations.

Specific Conclusions

On the basis of the findings of this investigation, for the population tested and within the limits of this study, the following conclusions were drawn:

1. Examinees from 16 to 18 years of age had a significantly higher level of understanding of selected basic arithmetical principles and generalizations than did examinees from 13 to 15 years of age.

2. Examinees from 22 to 24 years of age had a significantly higher level of understanding of selected basic arithmetical principles and generalizations than did examinees 25 to 30 years of age.

3. Examinees from 43 to 48 years of age had a significantly higher level of understanding of selected basic arithmetical principles and generalizations than did examinees 49 years of age or older.

4. Examinees with one to two years of teaching experience had a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees with three to five years of teaching experience.

5. Examinees in grade 12 had a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees in grade 8.

6. Examinees in grade 12 had a significantly higher level of understanding of selected basic arithmetical principles and generalizations than did examinees in grade 14 on a teacher preparatory program.

7. Examinees in grade 15 on a teacher preparatory program had a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees in grade 14 on a teacher preparatory program.

8. Examinees in grade 16 on a teacher preparatory program had a higher level of understanding of selected basic arithmetical principles

and generalizations than did in-service teachers with a bachelor's degree and no graduate work.

9. Teachers with a bachelor's degree, plus additional graduate work, had a higher level of understanding of selected basic arithmetical principles and generalizations than did teachers with a bachelor's degree and no graduate work.

10. Teachers with a master's degree, plus additional graduate work, possessed a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees with a master's degree and no additional graduate work.

11. Examinees with one or two semesters of study of mathematics beyond eight years of study of arithmetic had a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees with only eight years of study of arithmetic.

12. Examinees with five or six semesters of study of mathematics beyond eight years of study of arithmetic had a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees with three or four semesters of study of mathematics beyond eight years of study of arithmetic.

13. Examinees with seven or eight semesters of study of mathematics beyond eight years of study of arithmetic had a higher level of understanding of selected basic arithmetical principles and generalizations than did examinees with five or six semesters of study of mathematics beyond eight years of study of arithmetic.

General Conclusions

On the basis of the findings and within the limits of this study, the following general conclusions were drawn:

1. There seemed to be no consistency in the findings as to the significance of age as a factor contributing to examinees' understanding of selected basic arithmetical principles and generalizations.

2. Teaching experience did not seem to contribute significantly to examinees' understanding of selected basic arithmetical principles and generalizations.

3. Increased academic preparation seemed to contribute to examinees' understanding of selected basic arithmetical principles and generalizations.

4. Semesters of high school mathematics studied seemed to have a direct positive relationship to examinees' understanding of selected basic arithmetical principles and generalizations.

Implications

Relative to the findings and conclusions of this study, the following implications seem to be appropriate:

1. Some factor or factors affect examinees' understanding of selected basic arithmetical principles in three of the age groups.

2. The more formal high school mathematics that is studied the more pronounced is the relationship to understanding of basic arithmetical principles and generalizations.

3. The first two years of college work do not have a significantly positive relationship on examinees' understanding of basic arithmetical principles and generalizations. There seemed to be some factor or factors in the experiential background of examinees in grade 12 that significantly contributed to examinees' understanding of basic arithmetical principles and generalizations, a factor that was not present, at least to the same degree, in the experiential background of the examinees in grade 14.

4. Some factor or factors in the examinees' block of professional courses may have contributed to their understanding of basic arithmetical principles and generalizations.

5. Student teaching does not seem to contribute to examinees' understanding of selected basic arithmetical principles and generalizations.

6. For examinees on an elementary teacher preparatory program, some factor or factors rather consistently affect positively their understanding of selected basic arithmetical principles and generalizations.

7. The number of years away from formal college study may have a negative relationship to understanding of selected basic arithmetical principles and generalizations.

8. Course work is positively related to teachers' understanding of selected basic arithmetical principles and generalizations.

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APPENDIX

Appendix A

Personal Data Sheet

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Lewis W. Stoneking

PERSONAL DATA SHEET

What is your present age in years? _____

Are you _____ or _____? (Check one)
male female

Are you in grade ___ ___ ___ ___ ___ ___ ___ ___ ? (Check one)
8 12 14 16 AB AB+ MA MA+

Check the mathematics courses that you have taken or are taking in high school. (Check the proper number of semesters.)

Name of course	One semester	Two semesters	Three semesters	Four semesters
Gen. Math.				
Business Arith.				
Algebra				
Plane Geometry				
Solid Geometry				
Trigonometry				
_____ any others				

Check the mathematics courses that you have taken or are taking as undergraduate work in college. (Check the proper number of semesters.)

Name of course	One semester	Two semesters	Three semesters	Four semesters
Arith. Methods				
General Arith.				
Remedial Arith.				
College Algebra				
Plane Geometry				
Solid Geometry				
Trigonometry				
Analytical Geom.				
Calculus				
_____ any others				

How many years have you taught? ___ ___ ___ ___ ___ ___ ___ ___ ___
1 2 3 4 5 6 7 8 more

On which grade level have you had the most experience in teaching arithmetic?

___ ___ ___ ___ ___ ___ ___ ___ ___
Kg 1 2 3 4 5 6 7 8

Appendix B

Final Form of the Test of Basic Arithmetical Principles and Generalizations

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Test of Basic Arithmetical Principles and Generalizations

This is a test to see how well you understand arithmetic. It is not necessary to do any written work to select the correct responses.

Fill in the part of your answer sheet that has the name, date, age date of birth, and so on. Circle "M" or "F" in the corner. Do not fold or crease the answer sheet in any way.

Fill in the appropriate blanks on the Personal Data Sheet.

DIRECTIONS: Read each question and its numbered answers. When you have decided which answer is correct, blacken the corresponding space on the answer sheet with the special pencil. Make your mark as long as the pair of lines, and move the pencil point up and down firmly to make a heavy black line. If you change your mind, erase your first mark completely. Make no stray marks; they may count against you. Be sure that the space you mark on the answer sheet is numbered the same as the question. Do <u>not</u> mark on the test booklets.

SAMPLE: What is the sum of 4 and 3?

1. 12
2. 7
3. 1
4. "1," "2," and "3" are correct.

The second answer, answer "2," is correct, so you would mark your answer sheet with a heavy black mark under the second choice.

It is not necessary to do any written computation to determine the correct response. Omit those items that you could answer only by pure guess, but answer all you think you know even if you are not quite certain. Your score will be the number you have correct. <u>DO NO WRITTEN WORK TO FIND THE CORRECT RESPONSES</u>. Try each item, but do <u>not</u> spend much time on any one of them. If you cannot find the correct response, go on to the next item. There is no time limit on the test. Only one response is correct for each item.

In some of the items symbols such as , , and , are used to replace the familiar symbols 0, 1, . . ., 9. All laws, concepts, and relationships that hold in the present Hindu-Arabic (decimal) system of numeration and computation hold with the new symbols.

You may go through the entire test without stopping.

REMEMBER - <u>DO NO WRITTEN WORK TO FIND THE CORRECT RESPONSES</u>.

Now begin at the top of the next page.

Test of Basic Arithmetical Principles

1. Which of the following numerals represents the largest number?

   1. 7,564
   2. 7,546
   3. 7,645
   4. 7,654

2. Which of the following three place numerals represents the largest number?

   1. 432
   2. 4231
   3. 4321
   4. 342

3. Which of the following numerals has the digit 8 in the ones place?

   1. 4,168
   2. 1,684
   3. 6,841
   4. 8,416

4. Which of the following numerals has the largest digit in the thousands place?

   1. 457,326,158
   2. 624,182
   3. 8,123
   4. 999

5. Which of the following numerals represents a number 100 greater than 999,999?

   1. 1,000,099
   2. 1,000,999
   3. 1,009,099
   4. 1,999,999

6. How many sets of ten objects can be made of 7,834 objects

   1. 3
   2. 78
   3. 783
   4. 7830

7. In the numeral 7,777 the digit in the hundreds place represents a number how many times as great as the number represented by the digit immediately to the right of it?

   1. $\frac{1}{100}$
   2. $\frac{1}{10}$
   3. 100
   4. 10

8. In the number represented by the numeral 7,777 the 7 on the extreme left represents a number how many times as great as the number represented by the 7 on the extreme right?

   1. The 7 on the extreme left and the 7 on the extreme right represent the same number.
   2. Ten times as great
   3. 7000 times as great
   4. One thousand times as great

9. In the numeral 4,832 the digit in the hundreds place represents a number how many times as great as the number represented by the digit in the ones place?

   1. 400
   2. 4
   3. $\frac{1}{4}$
   4. $\frac{1}{400}$

10. Which of the following represents 15 hundreds and 7 ones?

    1. 157
    2. 1507
    3. 15,007
    4. Neither "1," "2," nor "3" is correct.

11. What is the reason for moving the second partial product one place to the left in the following example?

```
  483
X  52
 966
2415
25,116
```

    1. Because 50 times 3 equals 150
    2. Because 5 is the second digit in the multiplier
    3. Because it was taught that way
    4. Because the 5 is under the 8

12. What does 2320 represent in the following example?

$$\begin{array}{r} 580 \\ \times\ 43 \\ \hline 1740 \\ 2320 \\ \hline \end{array}$$

1. 2,320
2. 23,200
3. 232,000
4. 2,320,000

13. How many sets of 100 objects can be made of 1,000,000 objects?

1. 100,000
2. 10,000
3. 1,000
4. 100

14. The product of 875 X 284 would be how many more than the product of 815 X 284?

1. The problems would have to be computed to find out.
2. 60 tens X 284
3. 600 tens X 284
4. 6 tens X 284

15. What is the sum of this example?

$$\begin{array}{r} 9 \text{ tens and } 8 \text{ ones} \\ +\ 6 \text{ tens and } 5 \text{ ones} \\ \hline \end{array}$$

1. 1513
2. 163
3. 28
4. 153

16. What is the sum of the following example?

$$\begin{array}{r} 7 \text{ tens and } 6 \text{ ones} \\ +\ 12 \text{ tens and } 9 \text{ ones} \\ \hline \end{array}$$

1. 2115
2. 195
3. 205
4. 1915

17. In the following example when 3253 was multiplied by 3 it was necessary to "carry" or "substitute." What was "carried?"

$$\begin{array}{r} 3253 \\ \times\ \ \ 3 \\ \hline 9759 \\ \hline \end{array}$$

1. 1 one
2. 1 ten
3. 1 hundred
4. 1 thousand

18. How does the first partial product compare with the second partial product?

$$\begin{array}{r} 200 \\ \times\ 36 \\ \hline 1200 \\ 600 \\ \hline \end{array}$$

1. The first partial product is 2 times as great as the second partial product.
2. The first partial product is $\frac{1}{2}$ as great as the second partial product.
3. The first partial product is 5 times as great as the second partial product.
4. The first partial product is $\frac{1}{5}$ as great as the second partial product.

19. How does the second partial product compare with the final product?

$$\begin{array}{r} 5,827 \\ \times\ \ 25 \\ \hline \end{array}$$

1. 2 times as great
2. $\frac{4}{5}$ as great
3. 25 times as great
4. It would be necessary to compute the problem to find out.

20. Which of the following sets represents multiples of 10?

1. 10, 20, 30
2. 10, 15, 20
3. 35, 50, 30
4. 0, 100, 151

21. Which of the following statements would be true if two numerals "0" were annexed to the right of each of the numerals in this example?

$$\begin{array}{r} 72 \\ 46 \\ 941 \\ 713 \\ \hline \end{array}$$

1. The sum would be the same number.
2. The sum would be one thousand times as great.
3. It would have to be added both ways to find out.
4. The sum would be one hundred times as great.

22. How would the product be changed if the decimal point were moved one place to the left in both the multiplier (2.83) and the multiplicand (2835)?

$$\begin{array}{r} 2835 \\ \times\ 2.83 \\ \hline \end{array}$$

1. The product would be the same number.
2. The product would be one hundredth as great.
3. The product would be one tenth as great.
4. It would be necessary to compute the problem to find out.

23. How would the product be changed if the multiplicand were changed from 2,753 to 27,530 and the multiplier were changed from 38.42 to 3.842?

$$\begin{array}{r} 2{,}753 \\ \times\ 38.42 \\ \hline \end{array}$$

1. The product would be less.
2. The product would be the same number.
3. The product would be greater.
4. It would be necessary to compute the problem to find out.

24. What number is represented by the following notation?

$$2(10)^3 + 5(10)^2 + 9(10)^0$$

1. 259
2. 7,214
3. 2,590
4. 2,509

25. Which of the following sets represents powers of 10?

1. 10, 1000, 100
2. 30, 40, 50
3. 10, 20, 30
4. 100, 500, 1000

26. What is the base of the Hindu-Arabic system of numeration?

1. twenty
2. ten
3. twelve
4. one hundred

27. What is the sum of 6_8 and 3_8 in a numeration system with base eight?

1. 9_8
2. 18_8
3. 10_8
4. 11_8

28. What is the product of 4_5 and 3_5 in a numeration system with base five?

    1. 22_5
    2. 24_5
    3. 7_5
    4. 12_5

29. Which of the following represents all the numbers from one through ten in the Hindu-Arabic numeration system, when a numeration system with base eight is used?

    1. 1_8, 2_8, 3_8, 4_8, 5_8, 6_8, 7_8, 8_8, 9_8, 10_8
    2. 1_8, 2_8, 3_8, 4_8, 5_8, 6_8, 7_8, 8_8, 10_8, 11_8
    3. 1_8, 2_8, 3_8, 4_8, 5_8, 6_8, 7_8, 10_8, 11_8, 12_8
    4. 1_8, 2_8, 3_8, 4_8, 5_8, 6_8, 10_8, 11_8, 12_8, 13_8

30. In a closed number system with base two, which of the following represents the product $10_2 \times 10_2$?

    1. 0_2
    2. 10_2
    3. 4_2
    4. 100_2

31. In a number system with a base two, which of the following represents $1_2 + 1_2$?

    1. 2_2
    2. 10_2
    3. 100_2
    4. 4_2

32. Which of the following is correct?

    1. $5 + 0 = 0$
    2. $5 - 0 = 0$
    3. $5 \times 0 = 0$
    4. $5 \div 0 = 0$

33. Which of the following notations expresses "five and five thousandths?"

    1. 5.05
    2. .5005
    3. 5.050
    4. 5.005

34. Which of the following would have the same answer?

A	B	C	D	E
4.50	450	45.000	20.3	20.30
X 20.3	X .203	X 203	X 4.50	X 45.0

1. A, B, and C
2. D and E
3. A, B, and D
4. It is not possible to determine which answers would be the same until they are computed and compared.

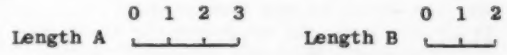
35. Which of the following represents the smallest number?

1. .08001
2. .038
3. .08
4. .1

36. Which of the following represents the largest number?

1. 143.600698
2. 143.0699
3. 143.606
4. 143.6

37. Which number expresses approximately the relationship of length B to length A?



1. 1.33
2. .75
3. 1.50
4. .67

38. How many tenths are approximately equivalent to .784?

1. 8 tenths
2. 78 tenths
3. .8 tenths
4. 784 tenths

39. How many hundredths are approximately equal to 273.5?

1. 273 hundredths
2. 2,735 hundredths
3. 27,350 hundredths
4. 273,500 hundredths

40. In the following example how would the product be changed if you moved the decimal point two places to the right in both the multiplier and the multiplicand?

$$\begin{array}{r} 2835 \\ \times\ 2.83 \\ \hline \end{array}$$

1. The product would be the same number.
2. The product would be ten thousand times as great.
3. The product would be one hundred times as great.
4. Since there is no decimal point in the multiplicand, it is not possible.

41. How would the product be affected if the zero were dropped from the multiplier in the following example?

$$\begin{array}{r} 8.295 \\ \times\ 27.50 \\ \hline \end{array}$$

1. The product would be ten times as great.
2. The product would be the same number.
3. The product would be one tenth as great.
4. Neither "1," "2," nor "3" is correct.

42. How would the sum in the following example be affected if the 687 were placed above the 18,423?

$$\begin{array}{r} 18{,}423 \\ +\ \ 687 \\ \hline \end{array}$$

1. The sum would be greater.
2. The sum would be less.
3. The sum would be the same number.
4. It is not possible to determine until the problems are computed and compared.

43. $3 + 2 =$

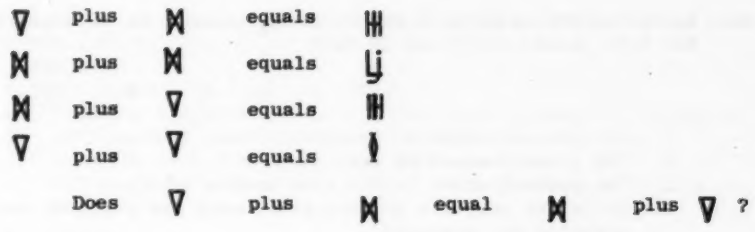
$3 + 3 =$

$2 + 2 =$

$2 + 3 =$

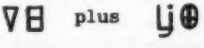
Does $2 + 3 = 3 + 2$?

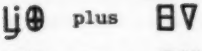
1. Never
2. Seldom
3. Sometimes
4. Always

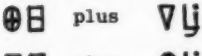
44. 

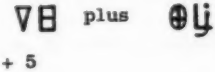
1. Never
2. Seldom
3. Sometimes
4. Always

45. Which of the following sums would be the same as the sum of

1. 

2. 

3. 

4. 

46. 5 + 4 = 4 + 5
6 + 2 = 2 + 6
3 + 9 = 9 + 3

Which of the following expresses the same principle?

1. 7 + 8 = 8 + 7
2. 7 + 8 = 15
3. 7 + 8 = 7 + 4 + 4
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

47. 4 X 3 = 3 X 4
5 X 6 = 6 X 5
2 x 8 = 8 X 2

Which of the following is <u>not</u> true?

1. $4\frac{1}{2}$ X 3 = 3 X $4\frac{1}{2}$
2. $6\frac{1}{2}$ X $9\frac{1}{4}$ = $9\frac{1}{4}$ X $6\frac{1}{2}$
3. $2\frac{1}{2}$ X 3 = $3\frac{1}{2}$ X 2
4. 15 X 21 = 21 X 15

48. How would the product in the following example be affected if the 687 were placed above the 18,423?

$$\begin{array}{r} 18{,}423 \\ \times \quad 687 \\ \hline \end{array}$$

1. The product would be greater
2. The product would be less
3. The product would be the same number
4. It is not possible to determine until the problems are computed and compared.

49. $4 \times 3 = 3 \times 4$
$2 \times 8 = 8 \times 2$
$5 \times 6 = 6 \times 5$

Which of the following expresses the same principle?

1. $7 \times 8 = 56$
2. $8 \times 7 = 7 \times 8$
3. $7 \times 8 = 7 \times 5 + 7 \times 3$
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

50. $2 + 3 = 10$
$4 + 2 = 11$
$10 + 4 = 14$
$3 + 1 = 4$

Which of the following is correct?

1. $2 + 3 + 4 = 14$
2. $3 + 1 + 4 = 8$
3. $4 + 4 + 2 = 19$
4. Neither "1," "2," nor "3" is correct.

51. Which of the following statements would <u>not</u> yield the correct answer to this example?

$$\begin{array}{r} 58 \\ +\ 43 \\ \hline \end{array}$$

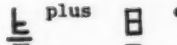
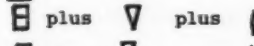
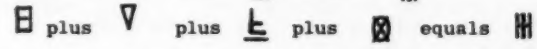
1. $3 + 8 + 40 + 50$
2. 4 tens + 5 tens + 8 ones + 3 ones
3. $50 + 3 + 8 + 40$
4. 50 tens + 8 ones + 40 tens + 3 ones

52. Which of the following statements would not yield the correct sum for 268 + 453?

1. 200 + 400 + 60 + 50 + 8 + 3
2. 3 ones + 8 ones + 5 tens + 6 tens + 4 hundreds + 2 hundreds
3. two hundreds plus six tens plus eight ones plus four hundreds plus five tens plus three ones
4. 200 hundreds + 60 tens + 8 ones + 400 hundreds + 50 tens + 3 ones

53. , and

Which of the following is true?

1. 
2. 
3. 
4. Neither "1", "2",nor "3" is correct.

54. 2 + 7 + 5 = 2 + (7 + 5) = 2 + 12 = 14, and (2 + 7) + 5 = 9 + 5 = 14
also
4 + 3 + 2 = 4 + (3 + 2) = 4 + 5 = 9, and (4 + 3) + 2 = 7 + 2 = 9

Which of the following expresses the same principle?

1. 8+3 + 5 = (8 + 3) + 5 = 11 + 5 = 16, and 8 + (3 + 5) = 8 + 8 = 16
2. 8 + 3 + 5 = 5 + 3 + 8
3. 8 + 3 + 5 = 16
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

55. 2 + 7 + 5 = 2 + (7 + 5), and (2 + 7) + 5 also
4 + 3 + 2 = 4 + (3 + 2), and (4 + 3) + 2

Which of the following expresses the same principle?

1. 8 + 3 + 5 = 16
2. 8 + 3 + 5 = 11 + 5 = 16, and 8 + 8 = 16
3. 8 + 3 + 5 = 5 + 3 + 8
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

56. Which of the following statements would not yield the correct answer to this example?

21 X 34 =

1. (1 one X 4 ones) + (1 one X 3 tens) + (2 tens X 4 ones) + (2 tens X 3 tens)
2. 1 X 34 + 2 X 34
3. 4 X 21 + 30 X 21
4. 1 X 34 + 20 X 34

57. 4 X 2 X 3 = (4 X 2) X 3 = 8 X 3 = 24, and
4 X (2 X 3) = 4 X 6 = 24 also
5 X 4 X 6 = (5 X 4) X 6 = 20 X 6 = 120 and
5 X (4 X 6) = 5 X 24 = 120

Which of the following expresses the same principle?

1. 2 X 5 X 6 = 2 X (5 X 6) = 2 X 30 = 60, and (2 X 5) X 6 = 60
2. 2 X 5 X 6 = 6 X 5 X 2
3. 2 X 5 X 6 = 60
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

58. 4 X 2 X 3 = (4 X 2) X 3 = 8 X 3, and 4 X (2 X 3) = 4 X 6 also
5 X 4 X 6 = (5 X 4) X 6 = 20 X 6, and 5 X (4 X 6) = 5 X 24

Which of the following expresses the same principle?

1. 2 X 5 X 6 = 10 X 6 = 60, and 2 X 30 = 60
2. 2 X 5 X 6 = 6 X 5 X 2
3. 2 X 5 X 6 = 60
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

59. The product 36 X 25 equals 36 X 20 plus which of the following?

1. 6 X 25
2. 5 tens times 36
3. 36 X 5
4. 5 X 30

60. Which of the following does <u>not</u> equal 5 times 797?

1. 800 X 5 minus 15
2. 797 X 10 ÷ 2
3. 700 X 5 plus 90 X 5 plus 7 X 5
4. 5 X 7 plus 5 X 9 plus 5 X 7

61. Which of the following is <u>not</u> correct for the product of 48 and 37?

1. (48 X 30) + (48 X 7)
2. (8 X 37) + (40 X 37)
3. (7 X 48) + (3 X 48)
4. (40 X 30) + (8 X 30) + (7 X 40) + (7 X 8)

62. Which of the following statements would <u>not</u> yield the correct answer to 45 X 3.4?

1. (4 tenths X 5 ones) + (4 tenths X 4 tens) + (3 ones X 5 ones) + (3 ones X 4 tens)
2. 3 X 45 + .4 X 45
3. 5 X 3.4 + 40 X 3.4
4. 4 X 45 + 3 X 45

63. Which of the following is correct for 19 X 27?

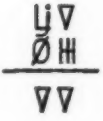
1. (20 X 27) plus (1 X 27)
2. (20 X 27) minus (1 X 27)
3. 2 X 10 X 27 - 19
4. It would be necessary to compute the answers to find out.

64. 7 X 9 = 7 X 5 + 7 X 4
8 X 21 = 8 X 20 + 8 X 1
9 X 6 = 6 X 6 + 3 X 6

Which of the following expresses the same principle?

1. 8 X 9 = 4 X 9 + 4 X 9
2. 8 X 9 = 72
3. 8 X 9 = 9 X 8
4. "1," "2," and "3" express the same principle.
5. Neither "1," "2," nor "3" express the same principle.

65.



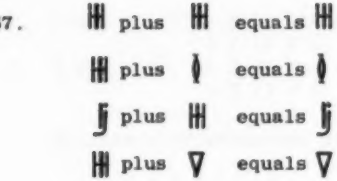
Which of the following is true?

1. plus equals
2. equals zero
3. equals zero
4. equals one

66.

Which of the following is true?

1. equals plus plus
2. equals plus plus
3. equals plus
4. equals

67.  plus equals

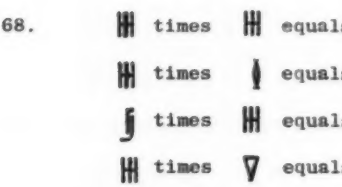
 plus equals

 plus equals

 plus equals

Which of the following statements is true?

1. equals one
2. equals zero
3. is more than
4. is less than

68.  times equals

 times equals

 times equals

 times equals

Which of the following statements is true?

1. equals zero
2. equals one
3. is more than
4. is less than

Appendix C

Summation Tables

TABLE 9. SUMMARY OF DATA FOR 1,066 EXAMINEES IN NINE AGE GROUP CLASSIFICATIONS

Age	Sum of scores	Sum of scores squared	Number of cases	Mean	Variance	Standard deviation
13-15	2,977	78,195	128	23.26	70.0	8.36
16-18	7,278	342,382	170	42.81	181.2	13.46
19-21	8,464	383,138	205	41.29	164.3	12.81
22-24	5,608	256,328	133	42.17	149.4	12.22
25-30	6,519	272,669	172	37.90	148.8	12.20
31-36	3,319	136,989	88	37.72	134.2	11.58
37-42	2,075	86,753	54	38.43	130.0	11.40
43-48	2,016	86,774	50	40.32	109.8	10.48
49 and older	2,165	80,193	66	32.80	139.0	11.78

TABLE 10. SUMMARY OF DATA FOR 437 EXAMINEES IN THREE TEACHING EXPERIENCE GROUP CLASSIFICATIONS

Years of experience	Sum of scores	Sum of scores squared	Number of cases	Mean	Variance	Standard deviation
1 or 2	4,514	200,720	111	40.66	154.5	12.43
3 to 5	4,856	198,706	132	36.79	152.0	12.33
6 or more	7,282	301,376	194	37.54	144.5	12.02

TABLE 11. SUMMARY OF DATA FOR 1,066 EXAMINEES IN NINE ACADEMIC CLASSIFICATIONS

Academic classification	Sum of scores	Sum of scores squared	Number of cases	Mean	Variance	Standard deviation
Grade 8	3,057	79,561	133	22.99	69.9	8.36
Grade 12	7,223	341,655	166	43.51	164.9	12.84
Grade 14	3,057	123,245	83	36.83	128.3	11.33
Grade 15	2,514	115,030	59	42.61	134.0	11.58
Grade 16	6,574	304,926	154	42.69	157.7	12.56
AB	3,007	121,761	86	34.97	193.3	13.90
AB+	11,392	481,508	294	38.75	136.4	11.68
MA	1,415	54,931	40	35.38	121.9	11.04
MA+	2,182	100,804	51	42.78	146.0	12.08

TABLE 12. SUMMARY OF DATA FOR 1,066 EXAMINEES IN EIGHT GROUP CLASSIFICATIONS ACCORDING TO THE NUMBER OF SEMESTERS OF MATHEMATICS STUDIED BEYOND GRADE 8

Semesters of mathematics	Sum of scores	Sum of scores squared	Number of of cases	Mean	Variance	Standard deviation
0	3,086	80,402	134	23.02	69.6	8.34
1 or 2	1,170	39,532	39	30.00	113.6	10.67
3 or 4	6,214	225,716	186	33.41	97.4	9.87
5 or 6	12,580	543,216	317	39.68	138.8	11.80
7 or 8	10,129	483,387	229	44.23	154.4	12.43
9 or 10	4,265	204,729	96	44.43	158.8	12.60
11 or 12	1,618	78,626	36	44.94	164.1	12.81
13 or more	1,359	67,813	29	46.86	142.3	11.93

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